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A Geometric Approach to Illustrate the Autocorrelation Effect in T^2 Control Chart of Hotelling

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Abstract A control chart is one of the main techniques of statistical control of process and using this technique means autocorrelation absence among the data of measured quality characteristics. The major objective of this article is to discuss the effect of the correlation and autocorrelation in Hotelling T^2 chart when there are two quality characteristics X and Y, whose autocorrelation and correlation structure is represented by a VAR(1) model. The study is done by using the Mahalanobis distance and a geometric approach with ellipses, results warn that the autocorrelation presence reduces the T^2 chart performance, restricting the chart ability to signal a special cause that is acting in the process. The higher degree of autocorrelation, the lower is the chart´s performance in detect assignable causes in the mean vector. Some examples are presented to illustrate the detrimental effect of T^2 chart when autocorrelation is present in the process.

Keywords Autocorrelation, Hotelling T^2 chart, VAR(1) model, Statistical process control

1. Introduction

One of the basic assumptions for control charts use is the independence among observations over time. However, in many cases, quality characteristic measures of neighboring items, according to the time they were produced, may present some dependence degree among the observations. This phenomenon is named autocorrelation. According to Mason and Young [1], many industrial operations of streaming present autocorrelation and one of the possible reasons is the gradual wear of critical components of process. Kim et al. [2] argue that independence hypothesis among observations of a variable can be violated by the high production rates that generate correlation and dependence among observations of neighbor products according to the manufacturing instant.

Autocorrelation affects the performance of traditional control charts [3]. Recent studies present alternatives to monitor these types of processes. Du and Lv [4] proposed a control chart based on the minimum euclidean distance to detect deviations on the average in autocorrelated processes. Lin et al. [5] considered the economic design of control chart ARMA, used in autocorrelated processes. Franco et al. [6] used the AR(1) model to describe the oscillatory behavior of the average and showed the optimal parameters of $\overline{X}$ chart using the Duncan model. Costa and Machado [7] also used the AR(1) model to investigate the effect of oscillatory behavior of the average in the performance of $\overline{X}$ chart with double sampling. In order to reduce the negative effect of autocorrelation on the $\overline{X}$ chart performance, Costa and Castagliola [8] presented a technique of systematic sampling called s-skip.

Modern systems with advanced technology and high production rates generated complex processes that are multidimensional, that is, many quality characteristics are measured and controlled. Pan and Jarrett [9] describe some of these processes that may have correlated and autocorrelated observations.

Hotelling [10] suggested the use of T^2 statistics for monitoring the mean vector in multivariate processes. Seven decades later, Chen [11] and Chen and Hsieh [12] showed the adaptive schemes that improve the performance of the T^2 chart. According to Hwarng and Wang [13], autocorrelation increases the false alarms rate, while correlation decreases the control graph power. The combined activity of autocorrelation and correlation in the performance of T^2 chart is worthy of investigation.

This article aims to graphically evaluate the effect of autocorrelation in two characteristics of measurable quality X and Y when there is correlation between observations of X and Y. The VAR(1) model was adopted to represent the structure of correlation and autocorrelation. It was considered in evaluating that the shift in the mean is the most important in the whole process and that the mean vector and

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the covariance matrix are known or estimated accurately.

The paper is organized as follows: section 2 describes the model that represents quality characteristics when there is autocorrelation in the process; section 3 presents some characteristics of the Hotelling T^2 chart; the effect of autocorrelation in bivariate processes is discussed and evaluated in section 4 and, finally, it is concluded about the work in section 5.

2. Autoregression Model and Cross-Covariance Matrix

The classical control procedures in multivariate processes consider the basic assumption that the observations follow a multivariate normal distribution and are independent, with mean vector $\boldsymbol{\mu}_0$ and variance-covariance matrix Σ.

$$\mathbf{X_t} = \boldsymbol{\mu}_0 + e_t \qquad t = 1,2,...,T \tag{1}$$

where: $\mathbf{X_t}$ represents observations by a vector of order p x 1 (p is the number of variables); e_t are independent random vectors of order p x 1 with multivariate normal distribution whose mean is zero and variance-covariance matrix Σ_e.

The independence assumption is violated in many manufacturing processes. Autoregression vectors of first order - VAR(1) are used to model multivariate processes with temporal correlation among observations of the same variable and correlation among observations of different quality characteristics [2, 3, 13-21] The VAR(1) model is represented by:

$$\mathbf{X}_t - \boldsymbol{\mu} = \Phi(\mathbf{X}_{t-1} - \boldsymbol{\mu}) + \boldsymbol{\varepsilon}_\mathbf{t} \tag{2}$$

being $\mathbf{X}_t \sim \mathrm{N_p}(\boldsymbol{\mu}, \Gamma)$ the vector of observations of dimension $(p \times 1)$ at instant t (p is the number of variables), $\boldsymbol{\mu}$ is the mean vector, $\boldsymbol{\varepsilon}_\mathbf{t}$ is a random vector with independent observations and multivariate normal distribution with zero mean and covariance matrix Σ and Φ is a matrix with autocorrelation parameters of order $(p \times p)$.

According to Kalgonda and Kulkarni [3], the $\mathbf{X}_t$ cross-correlation matrix has the following property: $\Gamma = \Phi\Gamma\Phi' + \Sigma$. After some algebraic manipulation, it is possible to obtain the relationship:

$$Vec\ \Gamma = \left(I_{p^2} - \Phi \otimes \Phi\right)^{-1} Vec\ \Sigma \tag{3}$$

where $\otimes$ is the product operator of Kronecker and *Vec* is the operator that transforms a matrix into a vector by stacking its columns.

To study the effect of correlation and autocorrelation of T^2 chart, it was considered a bivariate process (p=2):

$$\Phi = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix};\ \Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{4}$$

From (2) and (3), it follows that:

$$\Gamma = \begin{pmatrix} \sigma_X^2 = \left(1-a^2\right)^{-1} & \sigma_{XY} = \rho\left(1-ab\right)^{-1} \\ \sigma_{XY} = \rho\left(1-ab\right)^{-1} & \sigma_Y^2 = \left(1-b^2\right)^{-1} \end{pmatrix} \tag{5}$$

3. T^2 control chart of Hotelling

The T^2 control chart of Hotelling is one of the most known control schemes to detect deviations in the mean of multivariate processes [10]. When the mean vector $\boldsymbol{\mu}_0 = (\mu_{01}, \mu_{02})$ and the matrix Σ are known, the T^2 monitoring statistics of Hotelling is represented by:

$$T^2 = \left(\mathbf{X} - \boldsymbol{\mu}_0\right)' \Sigma^{-1} \left(\mathbf{X} - \boldsymbol{\mu}_0\right) \tag{6}$$

With the process in control, $T^2 \sim \chi_p^2$, if occurs a special cause that induces change in the mean vector for $\boldsymbol{\mu}_1 = (\mu_{11}, \mu_{12})$, $T^2 \sim \chi^2_{(p,\lambda)}$ with non-centrality parameter $\lambda^2 = \boldsymbol{\delta}'\Gamma_{\overline{X}}^{-}\boldsymbol{\delta}$, being $\boldsymbol{\delta} = (\delta_1 = \mu_{11} - \mu_{01}, \delta_2 = \mu_{12} - \mu_{02})$, see Wu and Makis [22]. When the mean vector and the covariance matrix are unknown and must be estimated, the control limits are calculated according to the monitoring phase [23].

Some authors use the non-centrality parameter (λ^2) as a displacement measure in the process mean vector [24, 25, 26, 1]. In this case, the chart performance is measured by:

$$ARL = \left\{1 - \left[\Pr\left(\chi^2_{(p,\lambda)}\right) < LC\right]\right\}^{-1} \tag{7}$$

where: ARL is the average number of samples up to the signal; CL is the control limit of the T^2 chart. The ARL measures the average number of samples until the occurrence of a false alarm if $\lambda^2 = 0$. When the process is out of control, the chart with the smallest ARL detects faster a process change.

4. Autocorrelation Effect in Bivariate Processes

The Hotelling T^2 chart was designed to be used when the independence hypothesis among observations of one or more quality characteristics is not violated. Excluding the effect of this hypothesis is quite detrimental to the control chart performance. To study the autocorrelation effect, it was considered the distance from $\mathbf{X}$ vector to the μ mean vector called Mahalanobis distance [27]. The Mahalanobis distance is given by:

$$D^2 = \left(\mathbf{X}-\boldsymbol{\mu}_0\right)' \Sigma^{-1} \left(\mathbf{X}-\boldsymbol{\mu}_0\right) \tag{8}$$

Ratio between cross-covariance matrix (Γ) and the elements of matrices Φ and Σ is calculated using equation (3). In presence of autocorrelation and correlation, the Mahalanobis distance is given by:

$$D^2 = \left(\mathbf{X}-\boldsymbol{\mu}_0\right)' \Gamma^{-1} \left(\mathbf{X}-\boldsymbol{\mu}_0\right) \tag{9}$$

Without generality loss, considering bivariate case where $\Phi = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ and

$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, when $\boldsymbol{\mu_0} = \left(\mu_{01} = 0; \mu_{02} = 0\right)$ and the vector $\mathbf{X} = \left(x; y\right)$ the distance D^2 is equivalent to:

$$D^2 = \frac{\left(-a^3bx^2 + 2\rho a^2b^2xy + a^2x^2 - 2\rho a^2xy - ab^3y^2 + abx^2 + aby^2 - 2\rho b^2xy + b^2y^2 - x^2 + 2\rho xy - y^2\right)}{\left(ab-1\right)^{-1}\left(-a^2b^2\rho^2 + a^2b^2 + a^2\rho^2 - 2ab + b^2\rho^2 - \rho^2 + 1\right)} \tag{10}$$

Equation (10) reveals influence of a , b e ρ in distance D^2.

If $a = b = 0$, namely, $\Phi = 0$ (there is no autocorrelation), the distance D^2 is reduces to:

$$D^2 = (\mathrm{x}^2\text{-}2\rho\mathrm{xy}+\mathrm{y}^2)\big/\left(1\text{-}\rho^2\right) \tag{11}$$

When there is no autocorrelation, that is, data are independent $D^2 \sim \chi^2_{(p;\alpha)}$. To evaluate the autocorrelation effect, it was used in this article the bivariate case and α = 0.01 ($\chi^2_{(p=2;\alpha=0.01)}$), where $D^2 = 10.5966$.

The performance of a control chart can be assessed in terms of number of samples that the chart uses to detect a shift in the characteristic to be monitored. When there is no displacement, the process is in statistical control. It is expected in this case the signal given by the chart to be a false alarm. The value $D^2 =$ 10.5966 amounts to a false alarm, on average, for each 200 samples evaluated when using the Hotelling T^2 chart [28, 29].

At the graphical evaluation of autocorrelation effect, it was considered that the displacement is type $\boldsymbol{\delta} = (\delta_1 = \mu_{11} - \mu_{01}, \delta_2 = \mu_{12} - \mu_{02})$, ie, the occurrence of a special cause shifts the mean vector $\boldsymbol{\mu_0} = \left(\mu_{01} = 0; \mu_{02} = 0\right)$ to a new level $\boldsymbol{\mu_1} = \left(\mu_{01} + \delta_{1\mu}; \mu_{02} + \delta_{2\mu}\right)$. In sections 4.1 and 4.2, there was the evaluation of effect autocorrelation of process in control ($\boldsymbol{\delta}$ = 0) and process out of control ($\boldsymbol{\delta} \neq 0$), respectively.

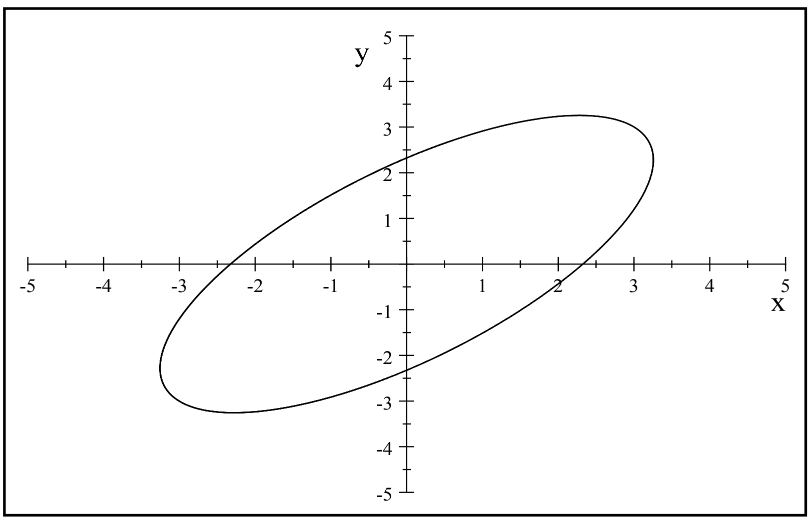

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Figure 1. Ellipse: $a = b = 0$ e $\rho = 0.7$

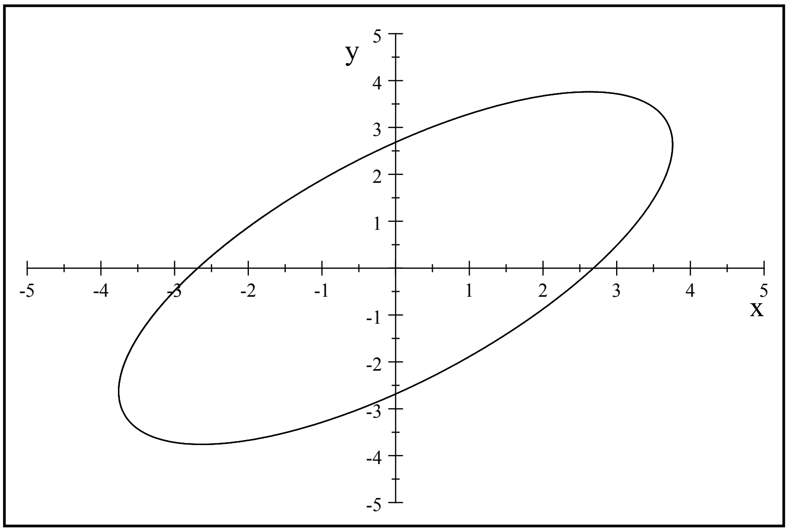

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Figure 2. Ellipse: $a = b = 0.5$ e $\rho = 0.7$

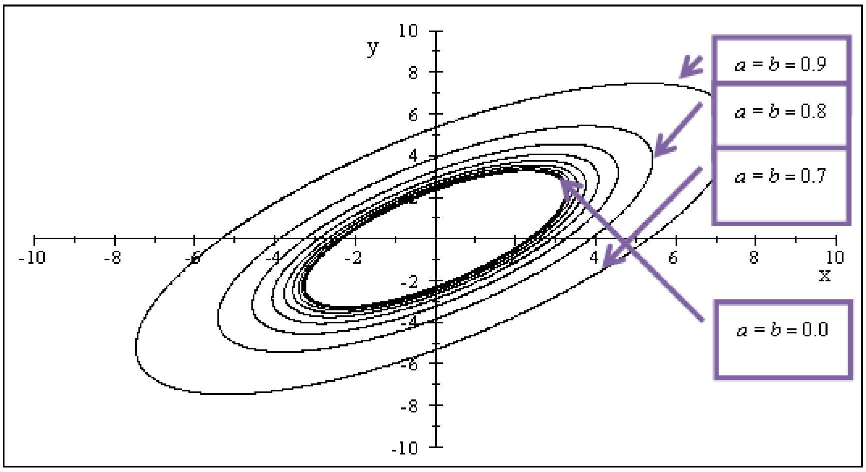

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Figure 3. Ellipses: a and $b \in \{0.0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9\}$ and $\rho = 0.7$

4.1. Graphical Evaluation of Autocorrelation Effect with Process in Control

In a process free of autocorrelation, $a = b = 0$ and $\rho =$ 0.7, it follows that $D^2 = 1.9608x^2 - 2.7451xy + 1.9608y^2$. The ellipse representing the distribution contour line for $D^2 = 10.5966$ is illustrated in Figure 1. Figure 2 illustrates the contour line for $a = b = 0.5$ and $\rho = 0.7$.

For a and $b \in \{0.0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9\}$, it can be observed in Figure 3 a graphic demonstration in which the greater the autocorrelation, the greater the elliptical area, that is, if the process is in control and there is autocorrelation in variables, it is necessary to adjust the graphical control limit otherwise, many false alarms will occur.

If the data are normally distributed, ellipses in Figure 3 represent all equidistant points, in Mahalanobis distance, from the origin. This suggests that all these points are equally likely to be governed by a multivariate normal distribution centered at (0,0), for $\boldsymbol{\mu}_0 = 0$. In T^2 Hotelling chart, the control limit (CL) equal to $D^2 =$ 10.5966, generates, generally, a false alarm every 200 samples collected when $a = b = 0$. Not so when $a = b \neq 0.0$, that is, the average false alarm rate does not correspond to an alarm every 200 samples collected, even if it is used as the CL value 10.5966. This means in practice that, when one uses the Hotelling T^2 chart, to consider the chart CL with chi-square distribution with p degrees of independence $\left(\chi^2_{(p)}\right)$ in autocorrelation presence will provide with a false alarms rate different than the desired.

4.2. Graphical Evaluation of Autocorrelation Effect with the Process out of Control

Figure 4 illustrates a zero-autocorrelation process with $a = b = 0$ and ρ = 0.7. The dashed ellipse with center (0,0) is a process in control and its equation is $D^2 = 1.9608x^2 - 2.7451xy + 1.9608y^2 = 10.5966$. The other ellipses represent the occurrence of a special cause that moves the mean vector towards a new baseline:

a) Displacement 1 → $\boldsymbol{\mu}_1 = (\mu_{01} + 1; \mu_{02} + 1)$; and:

$$D^2 = 1{,}9608x^2 - 2.7451xy - 1.1765x + 1.9608y^2 - 1.1765y + 1.1765 = 10.5966$$

b) Displacement 2 → $\boldsymbol{\mu}_1 = (\mu_{01} + 2; \mu_{02} + 2)$; and:

$$D^2 = 1.9608x^2 - 2.7451xy - 2.3529x + 1.9608y^2 - 2.3529y + 4.7059 = 10.5966$$

c) Displacement 3 → $\boldsymbol{\mu}_1 = (\mu_{01} + 3; \mu_{02} + 3)$; and:

$$D^2 = 1.9608x^2 - 2.7451xy - 3.5294x + 1.9608y^2 - 3.5294y + 10.588 = 10.5966$$

Figure 5 illustrates a process with autocorrelation with $a = b = 0.7$ and ρ = 0.7. The dashed ellipse with center in (0,0) is a process control and its equation is: $D^2 = x^2 - 1.4xy + y^2 = 10.06$. The value of 10.06 was used in order to make a fair comparison that, in autocorrelation presence, maintains the average false alarm rate equal to an alarm every 200 samples. As demais elipses representam a ocorrência de uma causa especial que desloca o vetor de médias para um novo patamar: The other ellipses represent the occurrence of a special cause that moves the mean vector towards a new level:

a) Displacement 1 → $\boldsymbol{\mu}_1 = (\mu_{01} + 1; \mu_{02} + 1)$; and:

$$D^2 = x^2 - 1.4xy - 0.18x + y^2 - 0.18y + 0.054 = 10.06$$

b) Displacement 2 → $\boldsymbol{\mu}_1 = (\mu_{01} + 2; \mu_{02} + 2)$; and:

$$D^2 = x^2 - 1.4xy - 0.36x + y^2 - 0.36y + 0.216 = 10.06$$

c) Displacement 3 → $\boldsymbol{\mu}_1 = (\mu_{01} + 3; \mu_{02} + 3)$; and:

$$D^2 = x^2 - 1.4xy - 0.54x + y^2 - 0.54y + 0.486 = 10.06$$

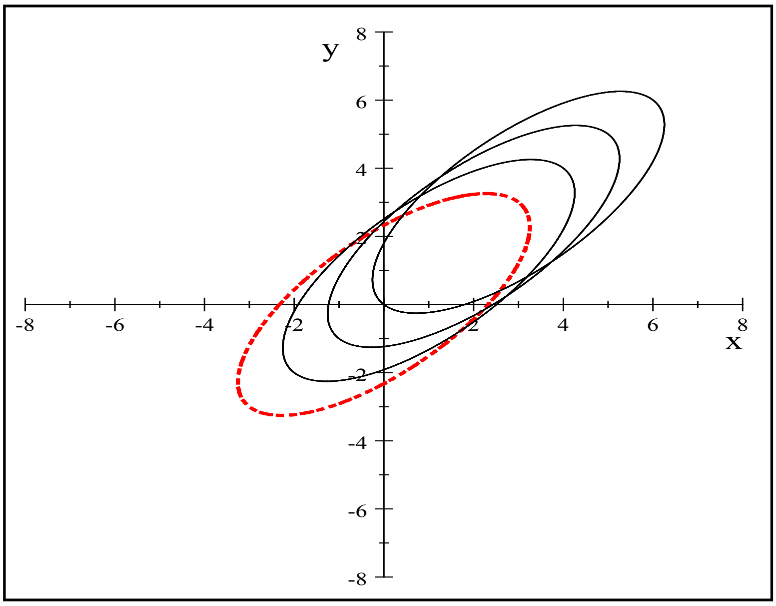

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Figure 4. Ellipses: $a = b = 0$ and $\rho = 0.7$

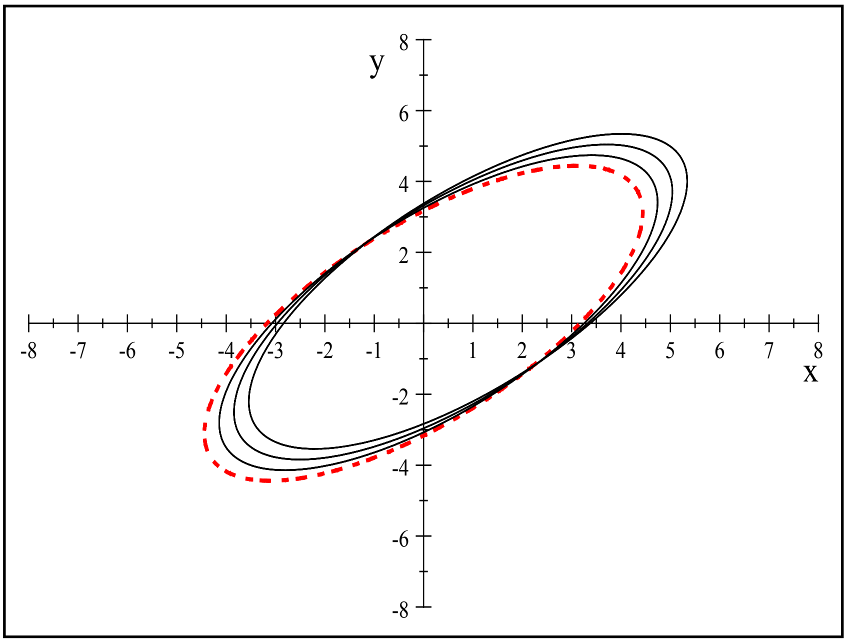

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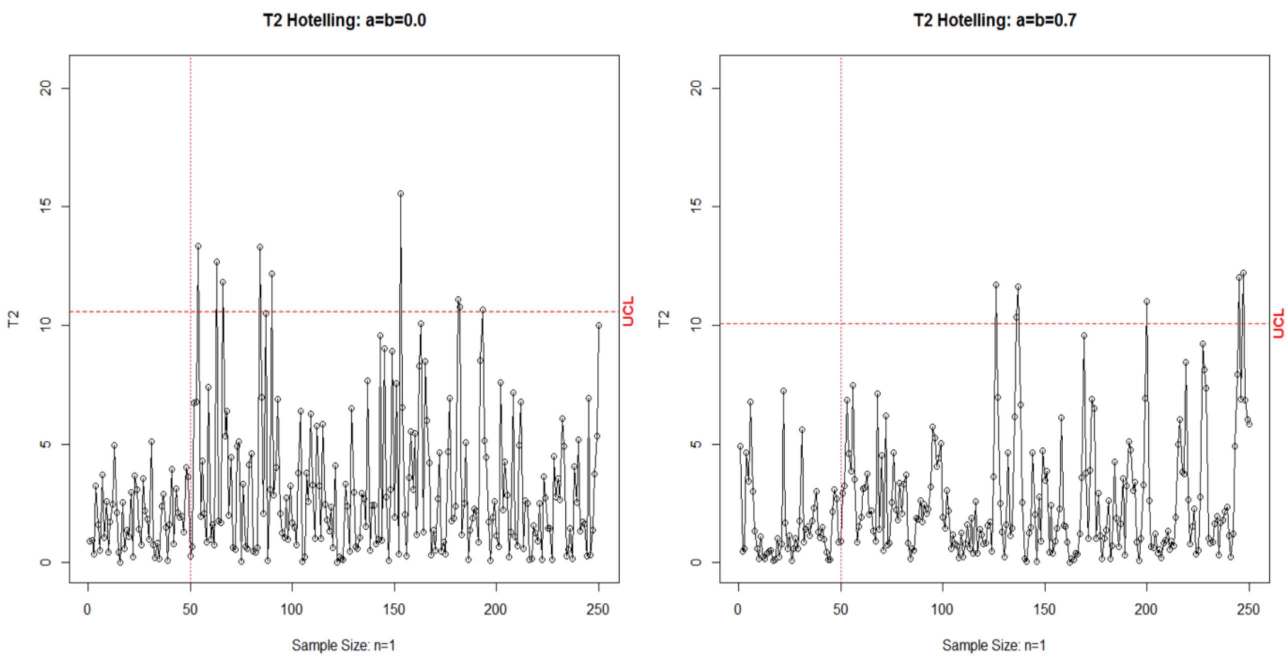
Figure 5. Ellipses: $a = b = 0.7$ and $\rho = 0.7$



Source: The authors.

Figure 6. Hotelling T^2 charts; $\boldsymbol{\delta} = (\delta_1 = 1.0\ ; \delta_2 = 1.0)$

From Figure 4, it is noted that in processes without autocorrelation the displacement in mean vector caused by a special question is represented by ellipses which depart from the center (0,0), indicating that the T^2 chart, in this case, presents superior performance in relation to the case where autocorrelation is present. In Figure 5, the ellipses have a higher resistance to remain close to the center (0,0) when occur displacements that disturb the mean vector, meaning that the T^2 chat performance is lower when the autocorrelation is present.

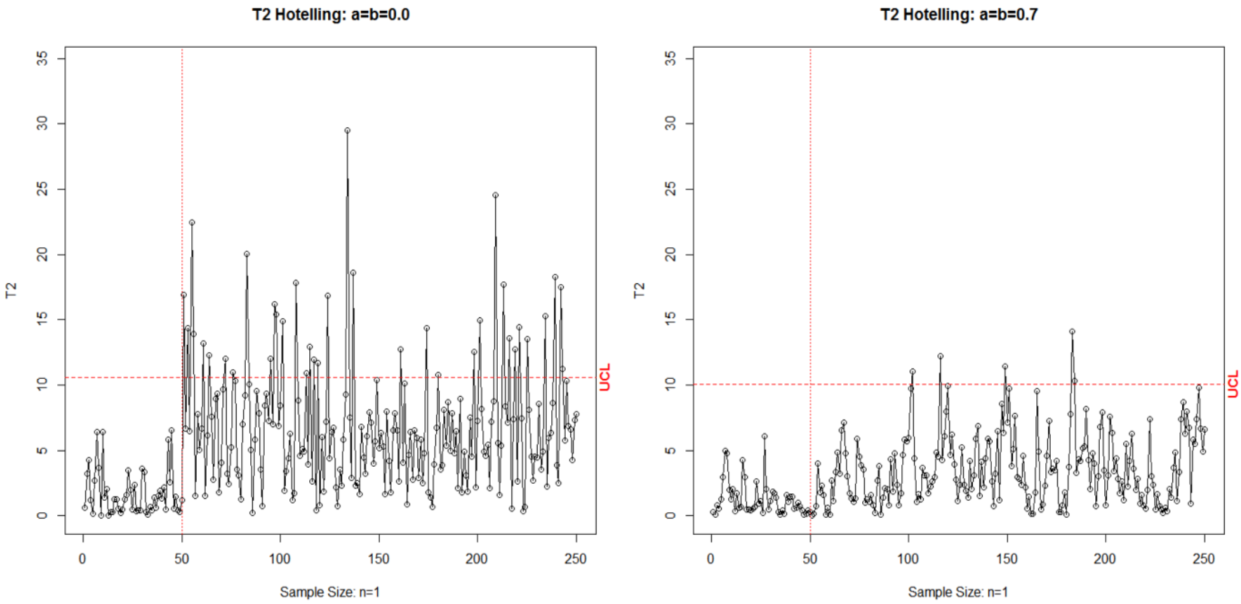


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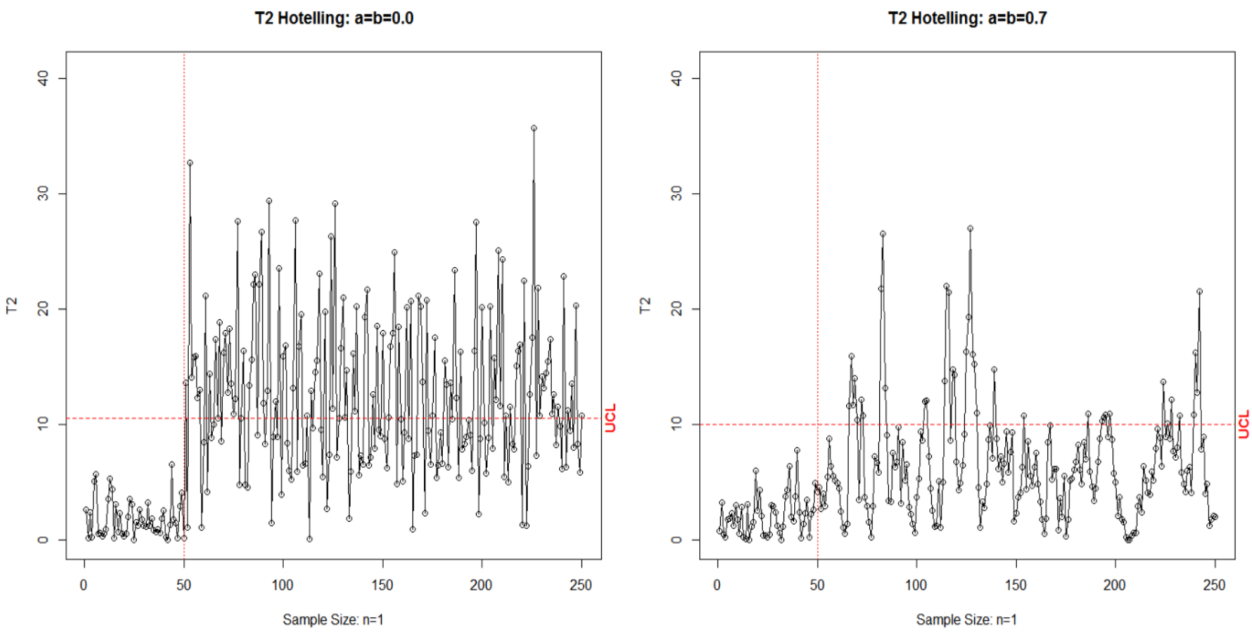
Figure 7. Hotelling T^2 charts; $\boldsymbol{\delta} = (\delta_1 = 2.0\ ; \delta_2 = 2.0)$



Source: The authors.

Figure 8. Hotelling T^2 charts; $\boldsymbol{\delta} = (\delta_1 = 3.0\ ; \delta_2 = 3.0)$

4.3. Example

Two types of bivariate processes were simulated and T^2 chart applied to control the variables of these processes. In the first case: $\boldsymbol{\mu}_0 = (0;0)$; $a=b$ = 0.0; ρ= 0.7 and CL=10.5966 (ARL_0=200). In the second case: $\boldsymbol{\mu}_0 = (0;0)$; $a=b$=0.7; ρ=0.7 e CL=10.06 (ARL_0=200). Three kinds of displacements were performed at the mean vector: $\boldsymbol{\delta} = (1.0;1.0)$, $\boldsymbol{\delta} = (2.0;2.0)$ and $\boldsymbol{\delta} = (3.0;3.0)$. Variables observations of the first and second processes were generated with the models of equation (1) and equation (2), respectively. Figures 6, 7 and 8 show the results. In each T^2 chart, the process has displacements in the mean vector from the sample 50. The results show that autocorrelation decreases the chart power to detect a special cause which operates in the mean vector of the process. Similar results were illustrated in Figures 4 and 5.

5. Conclusions

This article has evaluated the autocorrelation effect in a T^2 control chart since it is one of the most popular tools in academia and industry. The Mahalanobis distance, the same statistic used in the T^2 chart, was used to represent geometrically a process behavior in presence and absence of special causes that affect the average value of the monitored variables.

The autocorrelation hypothesis violation affects the T^2 chart performance, reducing the ability to detect deviations in the mean vector. The use of ellipses illustrated how the data of a process behave in presence of autocorrelation, that is, the masking displacement effect in the mean vector of variables. The displayed examples illustrated reduction that occurs in power T^2 chart to detect the presence of a special cause that shifts in the process mean vector. It is suggested, in future works, presentation of statistics or techniques that improve control chart in presence of autocorrelation.

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