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A Simple Procedure to Calculate the Control Limit of Z Chart

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Abstract A Z chartcan be used to monitor process quality characteristics. When there is correlation between observations of two measurable quality characteristics, X and Y, and there is dependence on the time among observations of X and also Y and this structure of correlation and autocorrelation is of a VAR(1) model, it is possible, for a certain false alarm rate, to relate the control limit of the Z chart with the variances and covariances of the cross-covariance matrix. This paper proposes a linear regression model to determine the control limit of Z chart. The method found in literature for obtaining the control limit of the Z chart is lower than the linear regression model proposed in this article; it is more complicated and does not guarantee the desired false alarm rate.

Keywords Z control chart, Linear regression, Autocorrelation

1. Introduction

The statistical process control helps managers understand, monitor and continuously improve the quality of products and services. In the 1930s, Shewhart [1] created the control charts to monitor processes, and then recognized the need to monitor processes considering the multivariate control.

The traditional control charts assume by means of hypothesis independence among observations of variable that one wishes to monitor. However, high production speeds generate correlation among the quality characteristics and dependence among observations of one quality characteristic of neighbor products according to the manufacturing instant [2]. Some studies were done in order to evaluate the charts performance of multivariate control in autocorrelation presence, concluding that there is a drop in these charts performance [3-6].

The multivariate processes monitoring whose observations are autocorrelated appears in recent publications. Mastrangelo and Forrest [7] have made available a program to generate autocorrelated data where it is possible to simulate displacement in value of average of variable under monitoring. Pan and Jarrett [8] proposed the use of waste of the VAR(p) model to monitor autocorrelated processes. The technique requires fitting the model to process data for later use of waste in the T^2 chart. Arkat [5] makes use of artificial neural networks for monitoring multivariate autocorrelated processes. Issam and Mohamad [6] propose the use of the SVR (support vector regression) method to monitor changes in the mean vector in autocorrelated processes through the MCUSUM control chart. Hwarng and Wang [9] established the use of neural networks that are able to identify shifts in the mean vector of autocorrelated processes. There are several other works on monitoring autocorrelated processes [10-13].

Autocorrelation compromises the use of control chart, for false alarms increase when it is disregarded, that is, when the control limits are established under hypothesis of autocorrelation absence [7, 14-16].

Kalgonda and Kulkarni [3] proposed the Z chart to monitor two or more quality characteristics for comments which follow a VAR(1) model. The advantage of Z chart in relation to T^2 chart is that it identifies the quality characteristic that suffers change in its average value. The authors present an empirical procedure to determine the control limit (CL) of Z chart. They assume that there is a correlation between the observations of X and Y and there is dependency in the time between X and Y observations and this correlation and autocorrelation structure is of a VAR(1) model. This article shows that for this correlation and autocorrelation structure there is a linear relationship between the CL of Z chart and variances and matrix covariance of cross-covariance of X and Y. For a wide range of values of cross-correlation and autocorrelations, it was obtained is a coefficient for determining the linear relationship model higher than 0.95.

This article aims to present a linear regression model to obtain the control limit of the Z chart that ensures the false alarm rate desired for a wide range of values of cross

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correlations and autocorrelations. The only method described in the literature for obtaining the CL was the Kalgonda and Kulkarni method [3] which, besides complicated, almost always gives control limits more spaced than it is necessary to meet the desired false alarm rate.

The paper is organized as follows: it is presented, in section 2, the model that describes the quality characteristics of a process with cross-correlations and autocorrelations; in section 3, the Z control chart; section 4 presents the regression model for obtaining the CL of Z chart and compares the false alarm rate calculated using the limits obtained by regression with the false alarm rate calculated using the limits obtained by the method of Kalgonda and Kulkarni [3].

2. Model Describing the Quality Characteristics

The classical control procedures in multivariate processes consider the basic assumption that observations follow a multivariate normal distribution and are independent with vector of means μ_0 and the variance-covariance matrix Σ_x.

$$X_t = \mu_0 + e_t \qquad t = 1, 2, ..., T \tag{1}$$

where X_t represents observations by a vector of order $px1$ ($p$is the number of variables); e_t are independent random vectors of order $px1$ with multivariate normal distribution whose mean is zero and variance-covariance matrix Σ_e.

The independence assumption is violated in many manufacturing processes, which makes equation (1) inadequate to represent such observations. Vector of auto regression of first-order, or VAR(1), equation (2), have been used to model multivariate processes with temporal correlation among observations of a variable and correlation among observations of different quality characteristics [2-6, 9, 17-20].

In multivariate autocorrelated processes, the VAR(1) model is represented as follows:

$$X_t - \mu_0 = \Phi(X_{t-1} - \mu_0) + e_t \tag{2}$$

where X_t is the data vector of order $px1$; μ_0 is the mean vector of order $px1$ and Φ is a matrix containing autoregressive parameters of order p x p and e_t are independent random vectors of order $px1$ with multivariate normal distribution whose mean is zero and variance-covariance matrix Σ_e.

If Φ is a zero matrix, equation (2) is reduced to equation (1), that is, one has the classical model for independent data over the time. Otherwise, the data will be dependent over the time and the model variation structure is represented by the cross-covariance matrix given by equation (3) [21].

Under the assumption that the process is stationary $E(X_t) = \mu_0$, for all t, cross-covariance matrix will be:

$$E\left[\left(X_t - \mu_0\right)\left(X_{t-h} - \mu_0\right)'\right] = \Gamma_x(h) \quad h = 0, 1, 2, ... \tag{3}$$

Being stationary means that μ_0 is constant for all X_t and the cross-covariance matrix does not depend on t, it depends only on h which represents the interval over the time and between the vector X_t and X_{t-h}.

The matrix $\Gamma_x(h)$ is formed by the elements $\gamma_{ij}(h)$ given by:

$$\gamma_{ij}(h) = E\left[\left(X_{it} - \mu_0\right)\left(X_{jt-h} - \mu_0\right)'\right] \quad i, j = 1, 2, ..., p \tag{4}$$

Since the cross-covariance matrix originally depends on the measurement unit of involved variables, sometimes its interpretation is not simple. A more convenient way to evaluate the relationship of variables in the process is given by using the cross-correlation matrix:

$$\rho_x(h) = D^{-1/2}\Gamma_x(h)D^{-1/2} \tag{5}$$

where D is the diagonal matrix formed by the elements $\gamma_{ij}(h)$, for all i=j, of matrix $\Gamma_x(h)$.

The cross-covariance matrix for h=0, $\Gamma_x(0)$, when Φ and Σ_e are known, can be obtained by the ratio of Yule-Walker [24]:

$$\Gamma_x(0) = \Phi\Gamma_x(0)\Phi' + \Sigma_e \tag{6}$$

Assuming X_t is a data vector with p-varied distribution and follow the model described in equation (2), according to Kalgonda and Kulkarni [3] and Kalgonda [20],

$$X_t \sim N_p\left[\mu_0; \Gamma_x(0)\right] \tag{7}$$

If the process is in statistical control, X_t follows a multivariate normal distribution with mean vector and cross-covariance matrix $\Gamma_x(0)$.

3. *Z* Control Chart

With the simultaneous use of *X* charts to control two or more quality characteristics, it is possible to identify which of them has been affected by the special cause. However, when the variables are dependent or correlated, to obtain the control limits of the *X* charts is no longer trivial [22], for the probability that the values of X_1, X_2, ... , X_p, are within the control limits is no more given by:

$$\left(1-\alpha\right)^p \tag{8}$$

where p is the number of variables and α the probability of a false alarm.

Kalgonda and Kulkarni [3] proposed a control chart called a Z chart for monitoring the mean vector of multivariate autocorrelated procedures. The chart maintains the overall error α and allows the variables identification whose means have changed with the emergence of a special cause. The authors adapted the statistical control technique of means vector for independent observations proposed by Hayter and Tsui [25] and considered that autocorrelation in the process follows the VAR(1) model.

At *t* time instant, the Z_t monitoring statistics of *Z* chart is given by $Z_t = Max_{1\le i\le p}\left[\left|Z_{it}\right|\right]$, where:

$$Z_{it} = \frac{X_{it}-\mu_{i0}}{\sqrt{\gamma_{ii,0(0)}}};\ \ i=1,2,...,p\ ;\ \ t=1,2,... \quad (9)$$

where X_{it} is the value of the *i-th* variable at instant of time *t* and $\gamma_{ii,0(0)}$ is the *i-th* diagonal element of the cross covariance matrix to *h*=0.

For a certain α value, the CL of the *Z* chart is given by:

$$\Pr\left[\left|Z_{it}\right| \le LC; i=1,2,...,p \left|\mu_i = \mu_0\right.\right] = 1-\alpha \quad (10)$$

The process is considered in statistical control if Z_t $\le$LC. Otherwise, there is evidence that the mean of at least one of the *p* variables changed.

The distribution of Z_t statistic is not known; Kalgonda and Kulkarni [3] obtained the CL by simulation following the steps:

- Step 1. Generating a large number of vectors (N = 10000) with observations according to the standard p-variate model$\left(X_t \sim N_p\left[\mu_0;\rho_X(0)\right]\right)$;
- Step 2: Calculating the Z_t statistic for each of the N vectors generated in step 1;
- Step 3 Obtaining the empirical distribution of the Z_t statistic, find the separatrix of order (1−α) and assign this value to the CL.

The steps described by Kalgonda and Kulkarni [3] almost always lead to control limits more widely spaced than necessary to meet the desired rate of false alarms (ARLo> $1/\alpha$). The ARLo is the average number of observations among false alarms. For independent and uncorrelated variables the ARLo = $1/\alpha$ [23].

To illustrate, let it be the bivariate case (*p*=2):

$$\mu_0 = (0,0)\ ;\ \Sigma_e = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix} \text{ and } \Phi = \begin{pmatrix} 0.7 & 0 \\ 0 & 0.7 \end{pmatrix},$$

then from (6) it has been obtained the cross-covariance matrix,

$$\Gamma_x(0) = \begin{pmatrix} \gamma_{11}(0)=1.9608 & \gamma_{12}(0)=0.9804 \\ \gamma_{21}(0)=0.9804 & \gamma_{22}(0)=1.9608 \end{pmatrix} \quad (11)$$

The method proposed by Kalgonda and Kulkarni [3] provides for $\alpha = 0.005$ a CL of 3.0191. For CL = 3.0191, it has been obtained by simulation one ARLo = 261.78. The appendix provides details of simulation.

Because of autocorrelation ARLo does not follow a geometrical distribution, for the probability α of false alarm is not constant. Depending on the parameters of the VAR(1) Model, the CL of Z chart provided by the method of Kalgonda and Kulkarni [3] leads to different ARLos. In order to solve this problem, this paper proposes a linear regression model which provides the CL of Z chart corresponding to the desired ARLo (see Figure 1 of section 4).

4. Proposed Method

In order to facilitate the use of the Z chart, the CL values were obtained by simulation for a wide range of parameter values of autocorrelation matrix and of the covariance matrix of bivariate VAR(1) error. Two regression models were made, one for ARLo of 200 and another for ARLo of 370. In regression models estimation, the CL values were allocated to the dependent variable vector and the elements values of cross-covariance matrix were allocated to the independent vectors matrix. The model fitted to the data providing R^2 values very close to 1, see Tables 2 and 4. Model parameters for ARLo of 200 and 370 are shown in Tables 1 and 3, respectively.

Table 1. Parameters of regression model - ARLo = 200

	Coefficient	*Standard error*	*ratio-t*	*p-value*
Constant	3.09844	0.00136	2279.78	<0.00001
$\gamma_{11}(0)$	-0.0311983	0.00078	-45.92	<0.00001
$\gamma_{22}(0)$	-0.0317356	0.00067	-47.38	<0.00001
$\gamma_{12}(0)$	-0.0451218	0.00136	-33.13	<0.00001

Table 2. Statistics of the model in Table 1

Statistics	Value
Sum of squared residuals	0.001
R-square	0.990
Statistical F (3.98)	3269.806
Standard regression error	0.003
Adjusted R-squared	0.990
P-value (F)	0.000

With the values of $\gamma_{11}(0)$, $\gamma_{22}(0)$ and $\gamma_{12}(0)$ of cross-covariance matrix it is possible to obtain the control limits of the Z chart.

For ARLo=200:

$$CL=3.09844- 0.0311983\,\gamma_{11}(0) - 0.0317356\,\gamma_{22}(0) - 0.0451218\,\gamma_{12}(0) \quad (12)$$

For ARLo=370:

$$CL= 3.26113- 0.0247597\,\gamma_{11}(0) -0.0247724\,\gamma_{22}(0) - 0.0337868\,\gamma_{12}(0) \quad (13)$$

It has also been considered the case where the ARLo is equal to 370. The results of regression model are shown in Tables 3 and 4.

Table 3. Regression model parameters - ARLo = 370

	Coefficient	*Standard Error*	*ratio-t*	*p-value*
Constant	3.26113	0.00249	1311.36	<0.00001
$\gamma_{11}(0)$	-0.0247597	0.00095	-25.99	<0.00001
$\gamma_{22}(0)$	-0.0247724	0.00096	-25.87	<0.00001
$\gamma_{12}(0)$	-0.0337868	0.00213	-15.83	<0.00001

Table 4. Statistics of model in Table 3

Statistics	Value
Sum of squared residuals	0.0169
R-squared	0.9466
Statistics F(3. 98)	738.1707
Standard error of regression	0.0116
Adjusted R-squared	0.9453
P-value(F)	0.0000

4.1. Sensitivity Analysis of the Proposed Method

To illustrate the use of the proposed method and its diagnosticability in the presence of autocorrelation, one considers a similar case of bivariate vector as presented in Kalgonda and Kulkarni [3]. The results in Figure 1 illustrate the ability of the proposed method to evaluate the CL. One considers the following scenarios to carry out the analysis:

ARLo is equal to 200, it has been adopted values *a* and *b* of matrix $\Phi = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ ranging from0.2 to 0.8 and values ρ of the matrix $\Sigma_e = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ equal to 0.3, 0.5 and 0.7. In practice these values are unknown; the elements of the cross-covariance matrix that depend on *a, b* and ρ are estimated according to equation (6). It is important to notice that an interesting subject of research is the study of Z chart in multivariate process.

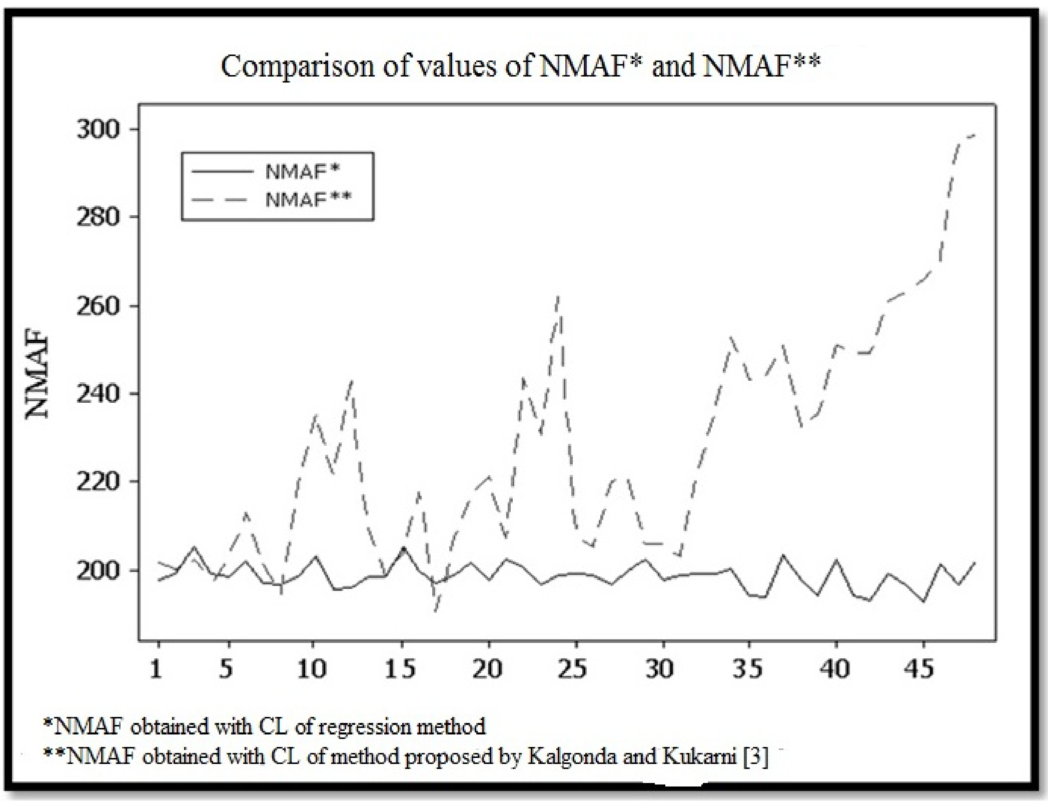

Figure 1. NMAF = ARLo obtained by regression and Kalgonda method [20]

The ARLo values for 48 different scenarios are presented in Table B1 of Appendix B and were used in the construction of Figure 1.

From Table B1 and Figure 1, it is observed that the regression model in order to obtain the CL is better than the Kalgonda and Kulkarni method [3], for it keeps the ARLo always close to 200 for all scenarios.

5. Conclusions

This paper has presented a method better than the one proposed by Kalgonda and Kulkarni [3] for obtaining the CL of Z chart. Better in order to provide control limits that lead to false alarms rates closer to those desired. The method of Kalgonda and Kulkarni [3] provides generally CL values larger than the one necessary; this excessive protection against false alarms occurrence reduces the control chart ability to detect changes in the process. The method proposed in this article requires great effort for the construction of the linear regression model; however, after obtaining it, the calculation of CL of Z chart is immediate.

Appendix A – Method Used in Simulation of Multivariate Temporal Series with Generation Process VAR(1)

Simulation of a multivariate temporal series with ***p*** dimension and ***T*** size:

1) It is created errors with Gaussian multivariate distribution of order ***p***, $e_t \sim N_p\left(0;\Sigma_e\right)$, by means of multiplication of matrix ***P*** of order (***p*** x ***p***) with vector ***V*** = ($v_1, \ldots, v_p$) of order (***px1***), where $PP^t = \Sigma_e$ and ***V*** ***~ N(0,1).***

$$e_t = P\begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} \quad \text{(A1)}$$

For instance, if p=2:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} p_{11} & 0 \\ p_{21} & p_{22} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \text{(A2)}$$

1) The step 1 is repeated ***T*** times for generation of a sort of errors.
2) With e_t values, it is obtained X_t in a recursive way by equation A3 turning ***t=1,2.....,T***.

$$X_t - \mu_0 = \Phi(X_{t-1} - \mu_0) + e_t \quad \text{(A3)}$$

where: X_t is a matrix of order***(px 1)***; μ_x is a mean matrix of order***(p x 1);*** Φ is the autocorrelation matrix of order***(p*** x ***p)***.

3) With vector generated in (3), it is obtained the statistics $Z_t = Max_{1\le i\le p}\left[\left|Z_{it}\right|\right]$.
4) The CL of ***Z*** chart is calculated by a binary search until the ARLo is equal to the desired value.

Appendix B –ARLo values

Table B1. Comparison of ARLo values based on regression model and on Kalgonda and Kulkarni method [3]

Scenarios	a	b	ρ	$\gamma_{11}(0)$	$\gamma_{22}(0)$	$\gamma_{12}(0)$	$CL_{Regression}$	ARLo*	$CL_{Kalgonda}$	ARLo**
1	0.2	0.2	0.3	1.0417	1.0417	0.3125	3.0188	198.10	3.0215	201.65
2	0.2	0.2	0.5	1.0417	1.0417	0.5208	3.0094	199.49	3.0097	200.45
3	0.2	0.2	0.7	1.0417	1.0417	0.7292	3.0000	205.25	3.0009	202.88
4	0.2	0.4	0.3	1.0417	1.1905	0.3261	3.0134	199.65	3.0158	197.93
5	0.2	0.4	0.5	1.0417	1.1905	0.5435	3.0036	198.35	3.0108	202.69
6	0.2	0.4	0.7	1.0417	1.1905	0.7609	2.9938	202.43	3.0061	213.29
7	0.2	0.6	0.3	1.0417	1.5625	0.3409	3.0010	197.18	3.0057	201.36
8	0.2	0.6	0.5	1.0417	1.5625	0.5682	2.9907	196.68	2.9870	194.75
9	0.2	0.6	0.7	1.0417	1.5625	0.7955	2.9805	198.36	3.0132	219.68
10	0.2	0.8	0.3	1.0417	2.7778	0.3571	2.9617	203.12	3.0076	235.66
11	0.2	0.8	0.5	1.0417	2.7778	0.5952	2.9509	195.74	2.9907	221.47
12	0.2	0.8	0.7	1.0417	2.7778	0.8333	2.9402	196.33	3.0107	242.68
13	0.4	0.2	0.3	1.1905	1.0417	0.3261	3.0135	198.33	3.0301	209.71
14	0.4	0.2	0.5	1.1905	1.0417	0.5435	3.0037	198.66	3.0053	198.65
15	0.4	0.2	0.7	1.1905	1.0417	0.7609	2.9939	205.38	2.9982	204.25
16	0.4	0.4	0.3	1.1905	1.1905	0.3571	3.0074	199.80	3.0314	217.69
17	0.4	0.4	0.5	1.1905	1.1905	0.5952	2.9967	197.14	2.9901	190.67
18	0.4	0.4	0.7	1.1905	1.1905	0.8333	2.9859	199.16	2.9878	207.48
19	0.4	0.6	0.3	1.1905	1.5625	0.3947	2.9939	201.52	3.0230	217.41
20	0.4	0.6	0.5	1.1905	1.5625	0.6579	2.9820	197.92	3.0185	220.98
21	0.4	0.6	0.7	1.1905	1.5625	0.9211	2.9701	202.76	2.9804	207.49

22	0.4	0.8	0.3	1.1905	2.7778	0.4412	2.9532	200.93	3.0212	243.71
23	0.4	0.8	0.5	1.1905	2.7778	0.7353	2.9400	196.50	2.9963	230.83
24	0.4	0.8	0.7	1.1905	2.7778	1.0294	2.9267	198.89	3.0177	262.15
25	0.6	0.2	0.3	1.5625	1.0417	0.3409	3.0013	199.24	3.0153	208.00
26	0.6	0.2	0.5	1.5625	1.0417	0.5682	2.9910	198.98	3.0061	205.37
27	0.6	0.2	0.7	1.5625	1.0417	0.7955	2.9807	196.82	3.0114	220.27
28	0.6	0.4	0.3	1.5625	1.1905	0.3947	2.9941	200.12	3.0242	220.99
29	0.6	0.4	0.5	1.5625	1.1905	0.6579	2.9822	202.53	2.9944	205.69
30	0.6	0.4	0.7	1.5625	1.1905	0.9211	2.9703	198.15	2.9806	205.98
31	0.6	0.6	0.3	1.5625	1.5625	0.4688	2.9790	198.97	2.9856	203.27
32	0.6	0.6	0.5	1.5625	1.5625	0.7813	2.9649	199.25	3.0041	221.97
33	0.6	0.6	0.7	1.5625	1.5625	1.0938	2.9508	199.27	2.9955	236.18
34	0.6	0.8	0.3	1.5625	2.7778	0.5769	2.9355	200.16	3.0096	253.04
35	0.6	0.8	0.5	1.5625	2.7778	0.9615	2.9182	194.19	2.9902	243.26
36	0.6	0.8	0.7	1.5625	2.7778	1.3462	2.9008	193.69	2.9761	244.32
37	0.8	0.2	0.3	2.7778	1.0417	0.3571	2.9626	203.38	3.0326	251.09
38	0.8	0.2	0.5	2.7778	1.0417	0.5952	2.9519	197.96	3.0037	232.80
39	0.8	0.2	0.7	2.7778	1.0417	0.8333	2.9411	194.47	3.0043	235.49
40	0.8	0.4	0.3	2.7778	1.1905	0.4412	2.9541	202.50	3.0319	250.93
41	0.8	0.4	0.5	2.7778	1.1905	0.7353	2.9408	194.15	3.0202	249.29
42	0.8	0.4	0.7	2.7778	1.1905	1.0294	2.9275	193.63	3.0086	249.46
43	0.8	0.6	0.3	2.7778	1.5625	0.5769	2.9362	199.41	3.0164	261.06
44	0.8	0.6	0.5	2.7778	1.5625	0.9615	2.9188	196.83	3.0142	263.45
45	0.8	0.6	0.7	2.7778	1.5625	1.3462	2.9014	193.16	2.9987	265.96
46	0.8	0.8	0.3	2.7778	2.7778	0.8333	2.8860	201.49	2.9850	270.41
47	0.8	0.8	0.5	2.7778	2.7778	1.3889	2.8610	196.47	2.9994	297.45
48	0.8	0.8	0.7	2.7778	2.7778	1.9444	2.8359	202.23	2.9755	298.72

* ARLo obtained with the CL of regression method (12).
** ARLo obtained with CL of method proposed by Kalgonda and Kulkarni [3].

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